2022

(Held in 2023)

BUSINESS MATHEMATICS

Paper : 1·3

Full Marks : 80

Time : Three hours

The figures in the margin indicate full marks for the questions.

Answer Question Nos. **1** and **2** and **any five** from the rest :

1. Choose the correct alternative : 1×10=10

   (a) The characteristics of the logarithm of the number 45.43 is

   (i) 1   (ii) 2

   (iii) 0   (iv) 3

   (b) If two rows or two columns of a determinant are identical the value of the determinants becomes

   (i) 0   (ii) 1

   (iii) 2   (iv) Can't say

(c) Find the 15th term of the series 3, 6, 9, 12, ...

    (i)   40   (ii)   42

    (iii) 43   (iv) 45

(d) The logarithm of 125 to the base $5\sqrt{5}$ is

    (i)   3   (ii)   2

    (iii) – 2   (iv) 4

(e) In an LPP

    (i)   only the objective function is linear

    (ii)   only the constraints are linear

    (iii) the objective function as well as the constrains are linear

    (iv) None of the above

(f) If A and B are disjoint sets then what is the value of $A \cap B$?

    (i)   0   (ii)   1

    (iii) ϕ (iv) None of these

(g) If $f(x) = 4x + 8$, $g(x) = 2x + 10$ and $h(x) = x + 2$ then which of the following is true ?

(i) $f(x) = g(x)$ at $x = 1$

(ii) $f(x) = h(x)$ at $x = 4$

(iii) $g(x) = h(x)$ at $x = 2$

(iv) None of the above

(h) $\log\left(\dfrac{a}{b}\right) + \log\left(\dfrac{b}{c}\right) + \log\left(\dfrac{c}{a}\right)$ equals to

(i) 1   (ii) −1

(iii) 0   (iv) 2

(i) If α and β be the roots of the equation $x(x - 3) = 4$ then the value of $\alpha^2 + \beta^2$ is

(i) 16   (ii) 17

(iii) 2   (iv) 8

(j) If TC, MC, AC and x represent the total cost, marginal cost, average cost and output respectively, then which of the following is/are true ?

(i) $\dfrac{d}{dx} TC = MC$

(ii) $\dfrac{TC}{x} = AC$

(iii) $\int MC dx = TC$

(iv) (i), (ii) and (iii)

2. Answer the following : *(any five)* 2×5=10

(a) Write *any two* properties of determinant.

(b) The third and 6th terms of a series in G. P. are 3 and 81 respectively. Find the first term and the common ratio.

(c) Define even function and odd function.

(d) Solve : $2x^2 - 13x + 15 = 0$

(e) Find the base about which logarithm of 64 is 4.

(f) Find the 7th term in the expansion of

$$\left(4x - \frac{1}{25x}\right)^{10}$$

(g) If $f(x) = \frac{1-x}{1+x}$, show that $f\left(\frac{1-x}{1+x}\right) = x$

3. (a) If $a^2 + b^2 = 7ab$ then prove that

$$2\log (a+b) = \log a + \log b + 2\log 3 \qquad 3$$

(b) If $x = 1 + 3^{2/3} + 8^{1/3}$, prove that

$$x^3 - 3x^2 - 6x - 4 = 0 \qquad 3$$

(c) If a, b, c are in G.P. and if p is the A.M. between a, b and q is the A.M. between b, c then prove that $\dfrac{a}{p} + \dfrac{c}{q} = 2$. 3

(d) If one root of the quadratic equation $x^2 - px + q = 0$ be twice the other, show that $2p^2 = 9q$. 3

4.   (a)   Evaluate :   2×4=8

   (i) $\lim\limits_{x\to 3}\left(\dfrac{x^2-5x+6}{x^2-4x+3}\right)$

   (ii) $\lim\limits_{x\to\infty}\left(\dfrac{2x^2+7x+5}{4x^2+3x-1}\right)$

   (iii) $\lim\limits_{x\to 2}\left(\dfrac{\sqrt{1+2x}-\sqrt{1-2x}}{x}\right)$

   (iv) $\lim\limits_{x\to 0}\left(\dfrac{a-\sqrt{a^2-x^2}}{x^2}\right)$

   (b)   The total cost $C(x)$ of producing x items is given by

   $C(x)=1000+5x$, when $0\le x\le 500$

   $\qquad\quad =200+4x$, when $500<x\le 2000$

   Discuss the continuity of $C(x)$ at $x=500$.   4

5.   (a)   Find the derivative $\dfrac{dy}{dx}$: *(any three)*

$$2 \times 3 = 6$$

   (i) $y = x^2 \log x$

   (ii) $y = (2x + 5)^4$

   (iii) $y = (x - 1)^2 (x + 2)$

   (iv) $y = \dfrac{1 + x}{1 - x}$

   (b)   The total cost $C(x)$ associated with producing and marketing x units of an item is given by

$$C(x) = .005x^3 - 0.02x^2 + 30x + 3000 \cdot$$

   Find —

      (i)   total cost when output is 4 units ;

      (ii)   average cost of output of 10 units ;

      (iii) marginal cost when output is 3 units.   6

6. (a) Solve the following LPP graphically : 6

Maximize $Z = 3x + 2y$

subject to

$2x + y \leq 100$

$x + y \leq 80$

$x \leq 40$

$x, y \geq 0$

(b) What do you mean by LPP ? Discuss the advantages and disadvantages of LPP. 6

7. (a) Expand the following : 3

$$\left(2 + 3x^2\right)^7$$

(b) Evaluate $\sqrt{99}$ to 6 places of decimals. 3

(c) Find the coefficient of x^8 in $\left(1 + x^2\right)^{10}$. 3

(d) Find the term independent of x in the expansion $\left(x + \dfrac{1}{x}\right)^9$. 3

8. (a) Find inverse of the matrix

$$\begin{pmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{pmatrix}$$

4

(b) Show that

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$$

4

(c) Solve by Cramer's rule

$$x + y + z = 3$$
$$2x - 3y + 5z = 4$$
$$x + 2y - 4z = -1$$

4

9. (a) The sum of three consecutive terms in A.P. is 54 and the product of the two extremes is 275. Find the terms. 4

(b) A man borrows Rs. 4,500 and promises to pay back in 30 instalments, each of value Rs. 10 more than the last. Find the first and last instalment. 4

(c) Find the sum up to n terms of the series

$$4 + 44 + 444 + \ldots\ldots$$

4

10. (a) If $f(x) = \dfrac{3x+2}{3x-2}$,

    show that $\dfrac{f(x)+1}{f(x)-1} = \dfrac{3x}{2}$ 3

(b) Of the 100 boarders of a hostel 80 drink tea, 40 drink coffee and 25 drink both tea and coffee. How many of them drink neither tea nor coffee ?   3

(c) Draw the graph of the function $y = |x|$.   3

(d) If $f(x) = \dfrac{|x|}{x}$ and c is any real number $(\neq 0)$ then show that

    $|f(c) - f(-c)| = 2$ 3

11. (a) Integrate the following :   3×3=9

    (i) $\displaystyle\int xe^x dx$

    (ii) $\displaystyle\int \dfrac{8x-10}{4x^2 - 10x + 8} dx$

    (iii) $\displaystyle\int_0^3 \left(x^2 - 3x + 2\right) dx$

(b) The marginal cost (MC) of a product is given by $MC = Rs\ (25 + 30x - 9x^2)$ and the fixed cost is known to be Rs. 550. Find the total cost function. 3

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