Total number of printed pages–7

3 (Sem–6) STS M 4

2021

STATISTICS

(Major)

Paper : 6·4

(Computer Programming and Multivariate Analysis)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

GROUP–A

(30 marks)

1. Answer the following questions as directed: 1×5=5

   (a) Let (X, Y) follows bivariate normal distribution with parameters $\left(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho\right)$. Then the conditional variance of $Y/X = x$ is ____________. *(Fill in the blank)*

Contd.

(b) State whether the following is acceptable as Fortran 77 statement.

$X + Y = Z$

(c) Let $(X, Y) \sim$ BVND $(0, 0, \sigma_1^2, \sigma_2^2, \rho)$. Then the pdf will be—

(i) $$f(x) = \frac{1}{2\pi\sigma_1\sigma_2\sqrt{(1-\rho^2)}} e^{-\frac{1}{2(1-\rho^1)}\left[\left(\frac{x}{\sigma_1}\right)^2 + \left(\frac{y}{\sigma_2}\right)^2 - 2\rho\frac{xy}{\sigma_1\sigma_2}\right]}$$

(ii) $$f(x) = \frac{1}{2\pi\sigma_1\sigma_2\sqrt{(1-\rho^2)}} e^{-\frac{1}{2(1-\rho^1)}\left[\left(\frac{x}{\sigma_1}\right)^2 + \left(\frac{y}{\sigma_2}\right)^2 + 2\rho\frac{xy}{\sigma_1\sigma_2}\right]}$$

(iii) $$f(x) = \frac{1}{2\pi\sigma_1\sigma_2\sqrt{(1-\rho^2)}} e^{-\frac{1}{2(1-\rho^1)}\left[\left(\frac{x}{\sigma_1}\right)^2 + \left(\frac{y}{\sigma_2}\right)^2 + 2\frac{xy}{\sigma_1\sigma_2}\right]}$$

(iv) None of the above

(Choose the correct option)

(d) Define Hotelling T^2 statistic.

(e) A flowchart is :

(i) Programming language

(ii) A graphical representation of an algorithm (*iii*) Step by step procedure of a programme written in English

(*iv*) None of the above

(Choose the correct option)

2. Answer the following questions : 2×5=10

(*a*) Write equivalent Fortran 77 statements of the following expressions:

(*i*) $e^{-x}x^{kx}$

(*ii*) $e^{-\left(\frac{x-a}{b}\right)^2}$

(*b*) Let $(X, Y) \sim \text{BVND}\left(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho\right)$. If $\rho = 0$, then prove that $X \sim N\left(\mu_1, \sigma_1^2\right)$ and $Y \sim N\left(\mu_2, \sigma_2^2\right)$.

(*c*) Let $\underset{\sim}{X} \sim N_3\left(\underset{\sim}{\mu}, \Sigma\right)$ where

$$\Sigma = \begin{pmatrix} 16 & -2 & 3 \\ -2 & 4 & 2 \\ 3 & 2 & 9 \end{pmatrix},$$

then find ρ_{13}.

(d) Given the sides of a triangle a, b, c. Write an algorithm to find its area

$$[\,Area = \sqrt{s(s-a)(s-b)(s-c)}, \quad where \quad 2s = a+b+c\,]$$

(e) Find $var\left(C\underset{\sim}{X}\right)$, where C is a $p \times p$ matrix of constant elements and X is a $p \times 1$ vector with variance-covariance matrix Σ.

3. Answer the following questions : 3×5=15

(a) Let $(X, Y) \sim$ BVND (0, 0, 1, 1, ρ), then prove (or disprove) that $X+Y$ and X–Y are independently distributed.

(b) Examine whether Hotelling T^2 is invariant under changes in the units of measurements.

(c) Derive bivariate normal density as a particular case of multivariate normal.

(d) Let $\underset{\sim}{X} \sim N_3\left(\underset{\sim}{\mu}, \Sigma\right)$.

Find the distribution of $\underset{\sim}{Y} = \begin{pmatrix} X_1 - X_2 \\ X_2 - X_3 \end{pmatrix}$

(e) Write a note on flowchart symbols and their uses.

GROUP–B

(30 marks)

Answer ***any three*** *questions from this Group :*

10×3=30

4. Derive the Characteristic function $\Phi(t)$ of the bivariate normal distribution (with usual parameters) and hence deduce the expression $\Phi(t)$ when variables are independent. 8+2=10

5. *(a)* Let $\underset{\sim}{X} \sim N_3\left(\underset{\sim}{\mu}, \Sigma\right)$ where

$$\Sigma = \begin{pmatrix} 1 & \rho & 0 \\ \rho & 1 & \rho \\ 0 & \rho & 1 \end{pmatrix}.$$

For what value of ρ, are $X_1+X_2+X_3$ and $X_1-X_2-X_3$ are statistically independent ? 5

(b) Let $\underset{\sim}{X} = N_p\left(\underset{\sim}{\mu}, \Sigma\right)$. Consider the transformation $\underset{\sim}{X} - \underset{\sim}{\mu} = C\underset{\sim}{Y}$, where C is $p \times p$ non-singular matrix and $\underset{\sim}{Y} = \left(y_1\ y_2 \cdots y_p\right)'$. Prove that the corresponding Jacobian of transformation is $|J| = |\Sigma|^{1/2}$. 5

6. *(a)* If $\underset{\sim}{X} \sim N_2(0, \Sigma)$, where

$\Sigma = (\sigma_{ij}),\ i, j = 1, 2,$ then prove that

$$\left(\underset{\sim}{X}' \Sigma^{-1} \underset{\sim}{X} - \frac{X_1^2}{\sigma_{11}}\right) \sim \chi_1^2$$ 8

(b) Assuming that $k = 1$, $l = 45$, $m = 7$, $n = 5$, evaluate the following Fortran 77 expressions : 2

(i) $(l + m) / n + k$

(ii) $l + m / n + k$

7. Let $\underset{\sim}{X} \sim N_3\left(\underset{\sim}{\mu}, \Sigma\right)$, where $\underset{\sim}{\mu} = (2, -3, 1)'$ and

$$\Sigma = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 3 & 2 \\ 1 & 2 & 2 \end{pmatrix}$$

Find the following :

(i) Conditional pdf of X_1 given X_2 and X_3.

(ii) Regression equation of X_1 on X_2 and X_3.

(iii) Conditional variance of X_1 given X_2 and X_3. 6+2+2=10

8. *(a)* Draw a flowchart indicating the steps to find the regression equation of Y on X. 5

(b) Given a random sample drawn from a population $\underset{\sim}{X} \sim N_p\left(\underset{\sim}{\mu}, \Sigma\right)$. Discuss the procedure to test the following hypothesis —

$\underset{\sim}{\mu} = \underset{\sim}{\mu_0}\ (specified)$, when Σ is known.

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