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(Held in 2021)

STATISTICS

(Major)

Paper : 5.1

(Sampling Distribution and Statistical Inference–I)

Full Marks : 42

Time : 2 hours

The figures in the margin indicate full marks for the questions

Group—A

(*Marks* : 21)

1. Answer the following questions as directed : 1×2=2

(a) What do you mean by 'asymptotic unbiasedness'?

(b) Range is order statistic.
(State True or False)

2. Answer the following questions briefly : 2×2=4

(a) Prove that Student's *t*-variate may be regarded as a particular case of Fisher's *t*-variate.

(b) State the necessary and sufficient condition for a distribution to admit sufficient statistic.

3. Answer any *three* questions from the following : 5×3=15

(a) If X is a chi-square variate with n degrees of freedom, then prove that for large n

$$\sqrt{2X} \sim N(\sqrt{2n}, 1)$$

(b) Find the maximum likelihood estimator of for the following probability distributions : 2+3=5

(i) $f(x, \ ) \quad e^{\ x};\ x \quad 0, \quad 0$

(ii) $f(x, \ ) \quad ^{x}(1 \quad)^{1 \ x};\ 0 \quad 1,\ x \quad 0 \text{ or } 1$

(c) Show that the transformation

$$w \quad \frac{\frac{\nu_1}{\nu_2}F}{1 \quad \frac{\nu_1}{\nu_2}F}$$

changes the *F*-distribution to the Beta distribution.

(d) What do you mean by a minimum variance bound (MVB) estimator? Let $x_1, x_2, x_3, \ldots, x_n$ be a random sample drawn from a normal population with mean zero and variance ^{2}. Find MVB estimator for ^{2}. 1+4=5

(e) Explain briefly the method of minimum chi-square.

GROUP—B

(*Marks* : 21)

4. Answer any *three* questions from the following : 7×3=21

(a) Let X follows t-distribution with k degrees of freedom. Then show that $\dfrac{1}{1 \quad \dfrac{X^2}{k}}$ follows Beta distribution.

(b) For the multinomial distribution

$$p(x_1, x_2, \ldots, x_k) \quad \frac{n!}{n_1!n_2!\ldots n_k!} p_1^{n_1} p_2^{n_2} \ldots p_k^{n_k}$$

where

$$n_1 \quad n_2 \quad \ldots \quad n_k \quad n$$

$$p_1 \quad p_2 \quad \ldots \quad p_k \quad 1$$

find the maximum likelihood estimator of p_i.

(c) With the help of an example for each case, show that—

(i) a biased estimator may be consistent;

(ii) an unbiased estimator may also be consistent.

(d) The sample values from a population with p.d.f. $f(x) \quad (1 \quad)x \ ; 0 \quad x \quad 1, \quad 0$ are given below :

0.46, 0.38, 0.61, 0.82, 0.59, 0.53, 0.72, 0.44, 0.59, 0.60

Find the estimate of by the method of moments.

(e) State few situations where one can use order statistic. Show that for random sample of size 2 from normal population $N(0, \ ^2)$, $E[X_{(1)}] \quad \frac{}{\sqrt{}}$, where $X_{(1)}$ is the first-order statistic. 2+5=7

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