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STATISTICS

(Major)

Paper : 5.1

(Sampling Distribution and Statistical Inference–I)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following questions as directed : $1 \times 7 = 7$

(a) Normal distribution is a particular case of chi-square distribution with

(i) n d.f.

(ii) $(n-1)$ d.f.

(iii) $(n-2)$ d.f.

(iv) None of the above

(Choose the correct option)

(b) Write down the p.d.f. of a single-order statistic.

(c) What is relative efficiency?

(d) The d.f. of a Fisher's t-statistic is 9. What will be the d.f. of the corresponding χ^2-statistic?

(e) State one application of F-statistic.

(f) Maximum likelihood estimators (MLEs) are not necessarily unbiased.

(State True or False)

(g) State factorisation theorem.

2. Answer the following questions : 2×4=8

(a) State two applications of order statistics.

(b) State the essentials of 'sufficient estimators'.

(c) Show that

$$F(n_1, n_2) = \frac{1}{F(n_2, n_1)}$$

where $F(n_1, n_2)$ represents F variate with n_1 and n_2 d.f.

(d) Show that the sample r-th moment is an unbiased estimator of population r-th moment, if it exists.

3. Answer any *three* of the following : 5×3=15

(a) Let $y_1 < y_2 < y_3$ be the order statistics of a random sample of size 3 from the uniform distribution having the density function

$$f(x;\theta) = \frac{1}{\theta}, \quad \begin{array}{l} 0 < x < \theta \\ 0 < \theta < \alpha \end{array}$$
$$= 0, \quad \text{elsewhere}$$

Show that $4y_1, 2y_2$ and $\frac{4}{3}y_3$ are all unbiased estimators of θ.

(b) When $\nu_1 = 2$, show that the significance level of F corresponding to a significant probability p is

$$F = \frac{\nu_2}{2}(p^{-\frac{\nu_2}{2}} - 1)$$

where ν_1 and ν_2 have their usual meanings.

(c) Let $x_1, x_2, \cdots, x_n$ be a random sample of n observations from the first kind of beta distribution with parameters α and β. Find the estimators of α and β by the method of moments.

(d) If the random variables X_1 and X_2 are independent and follow the χ^2-distribution with n d.f., show that

$$\frac{\sqrt{n}(X_1 - X_2)}{2\sqrt{X_1 X_2}}$$

is distributed as Student's t with n d.f. and independently of $X_1 + X_2$.

(e) Let $x_1, x_2, \cdots, x_n$ be a random sample of n observations from Cauchy population with parameter μ. Show that the Cramer-Rao lower bound of the variance of an unbiased estimator of μ is $\frac{2}{n}$, where n is the sample size.

4. Answer the following questions : 10×3=30

(a) Derive χ^2-distribution and state the applications of χ^2-statistic. 10

Or

What do you mean by 'minimum variance unbiased estimator (MVUE)? If T_1 and T_2 are two MVUEs of a parameter $\tau(\theta)$, each being of efficiency e, then show that the coefficient of correlation ρ between them satisfies the inequality

$$2e - 1 \leq \rho \leq 1$$

2+8=10

(b) State three applications of t-distribution. Show that the statistic

$$t = \frac{r}{\sqrt{1-r^2}}\sqrt{n-2}$$

is distributed as Student's t with $(n-2)$ d.f. under the null hypothesis $H_0 : \rho = 0$, r being sample correlation coefficient. 3+7=10

Or

Define r-th order statistic. Obtain the joint p.d.f. of $X_{(r)}$ and $X_{(s)}$, $r < s$ in a random sample of size n from a population with continuous distribution function $F(\cdot)$. Hence deduce the p.d.f. of sample range $W = X_{(n)} - X_{(1)}$. 2+5+3=10

(c) Obtain the asymptotic distribution of maximum likelihood estimator (MLE). 10

Or

Write a note on the 'method of minimum chi-square'. Find the MLEs of α and β for random sample drawn from the exponential distribution

$$f(x; \alpha, \beta) = y_0 \exp\{-\beta(x-\alpha)\}, \quad \alpha < x < \infty$$
$$\beta > 0$$

where y_0 is a constant. 3+7=10

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