2018

PHYSICS

(Major)

Paper : 3.1

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

GROUP—A

(Mathematical Methods)

(*Marks* : 25)

1. Answer the following questions : 1×3=3

(a) What do you mean by nilpotent matrix?

(b) What is the condition for a symmetric matrix to be a Hermitian matrix?

(c) What is unitary matrix?

2. Find the rank of the matrix

$$\begin{pmatrix} 1 & 2 & 0 \\ 2 & 4 & 0 \\ 4 & 8 & 0 \end{pmatrix}$$ 2

3. Answer any *two* of the following questions : 5×2=10

(a) *(i)* Prove that the trace of the product of a symmetric and an anti-symmetric matrix is zero. 2

(ii) Find the inverse of the matrix

$$\begin{pmatrix} 3 & -1 & 1 \\ -15 & -6 & -5 \\ 6 & -2 & 2 \end{pmatrix}$$ 3

(b) *(i)* What are proper and improper orthogonal matrices? 2

(ii) Prove that every non-singular square matrix has a unique inverse. 3

(c) *(i)* Show that every characteristic vector of a matrix has a unique characteristic root. 2

(ii) Find the matrix B such that $A = BC$, if

$$A = \begin{pmatrix} 2 & 3 & -2 \\ 4 & -1 & -2 \\ 0 & 1 & 0 \end{pmatrix} \text{ and } C = \begin{pmatrix} 1 & 2 & -1 \\ 2 & -1 & -1 \\ -1 & 2 & 1 \end{pmatrix}$$ 3

4. Answer *either (a)* and *(b) or (c)* and *(d)* : 5×2=10

(a) State and prove Cayley-Hamilton theorem. 5

(b) Find the eigenvalue and eigenvector of the matrix

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 2 & 3 \\ 0 & 0 & 2 \end{pmatrix}$$ 5

(c) If three matrices A, B and C are given by

$$A = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad B = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}$$

$$\text{and } C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

prove that $D^2 = A^2 + B^2 + C^2 = 2I$ 5

(d) Using schematic diagram, obtain the two-dimensional rotational matrix. 5

GROUP—B

(Electrostatics)

(*Marks* : 35)

5. Choose the correct answer/Answer the following questions : 1×3=3

(a) The relation $D = \varepsilon E$ is true for

(i) any medium

(ii) homogenous medium

(iii) isotropic medium

(iv) homogenous and isotropic media

(b) The induced surface charge q' is related to q as

(i) $q' = \frac{q}{k}$

(ii) $q' = q$

(iii) $q' = q\left(1 - \frac{1}{k}\right)$

(iv) $q' = q\,(1 - k)$

(where k is dielectric constant)

(c) The unit of electric potential in terms of base unit of SI is

(i) kgm^2S^{-1}

(ii) $kgm^2S^{-1}A^{-1}$

(iii) kgm^2S^{-2}

(iv) $kgm^2S^{-3}A^{-1}$

6. Answer the following questions : 2×3=6

(a) What do you mean by equipotential surfaces?

(b) If the electric field is given by $E = 8x + 4y + 3z$, calculate the electric flux through a surface of area 100 units lying in the x-y plane.

(c) What is the acceleration of a charged particle of mass m and charge q placed in an electric field E?

7. Answer any *two* of the following questions : 3×2=6

(a) Calculate the electrostatic energy of a system of charged particles.

(b) A sphere of radius R is connected by wire with a smaller sphere of radius r. If the spheres were charged with Q and q respectively, show that the electric field is higher at the surface of the smaller sphere.

(c) The potential of a certain charge configuration is expressed by $V = 2x + 3xy + y^2$ volt. Find the electric intensity at point (5, 2). What acceleration does an electron experience in the x-direction?

8. Answer any *two* questions : 10×2=20

(a) (i) Find an expression for the electric field intensity at an axial point of a charged disc. 5

(ii) What is the principle of 'method of images'? A charge Q is placed in front of an earthed conducting sphere of radius R. Calculate the potential and the field at a general point (r, θ). 5

(b) (i) Using Gauss' law, find an expression for electric field in a uniformly charged sphere. 5

(ii) Using Laplace's equation, obtain the expressions for potential and electric field intensity between two parallel planes. 5

(c) (i) State and prove the differential form of Gauss' law in dielectric. 5

(ii) Establish the Clausius-Mossotti relation using Laplace equations. 5

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