2018

PHYSICS

(Major)

Paper : 2.1

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

GROUP—A

(Mathematical Methods–II)

(*Marks* : 35)

1. Answer the following questions : 1×3=3

(a) Evaluate $\vec{a} \times \frac{d^2\vec{r}}{dt^2} = \vec{b}$, where $\vec{a}$ and $\vec{b}$ are constants.

(b) Define Laplacian in curvilinear coordinate system.

(c) Evaluate $\Gamma(-\frac{1}{2})$.

2. Find the value of $\iint\limits_S \vec{r}\cdot\hat{n}\,dS$, where S is closed surface. 2

3. Answer any *two* of the following questions : 5×2=10

(a) *(i)* Find the value of $\int\limits_C \vec{F}\times d\vec{r}$, where $\vec{F} = xy\hat{i} - z\hat{j} + x^2\hat{k}$ and C is the curve $x = t^2$, $y = 2t$, $z = t^3$ from $t = 0$ to $t = 1$. 3

(ii) If S be a closed surface and $\vec{r}$ denotes the position vector of any point (x, y, z) measured from origin O, then show that

$$\iint\limits_S \frac{\hat{n}\cdot\vec{r}}{r^3}\,dS = 0$$

when O lies outside the closed surface S. 2

(b) *(i)* Express the acceleration $\vec{a}$ of a particle in cylindrical coordinates. 3

(ii) Represent the vector $\vec{A} = z\hat{i} - 2x\hat{j} + y\hat{k}$ in cylindrical coordinates. 2

(c) (i) Evaluate $\int_0^\infty x^{n-1} e^{-h^2x^2}\,dx$. 3

(ii) Prove that $x\delta(x) = 0$. 2

4. Answer any *two* of the following questions : 10×2=20

(a) (i) Find the value of

$$\iint_S (\vec{\nabla} \times \vec{F}) \cdot \hat{n}\, dS$$

for $\vec{F} = (y - z + 2)\hat{i} + (yz + 4)\hat{j} - xz\hat{k}$, where S is the surface of the cube $x = y = z = 0$, $x = y = z = 2$ above the xy-plane. 5

(ii) If R is a closed region in the xy-plane bounded by a simple closed curve C, and M and N are continuous functions of x and y having continuous derivatives in R, then show that

$$\oint_C (M\,dx + N\,dy) = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) dx\,dy$$

where C is traversed in the positive direction. 5

(b) (i) Prove that

$$\iiint_V \vec{\nabla}\phi\, dV = \iint_S \phi\hat{n}\, dS$$ 5

(ii) If the normal surface integral of a vector point function $\vec{G}$ over every open surface is equal to the tangential line integral of another function $\vec{F}$ round its boundary, prove that $\vec{G} = \text{curl}\,\vec{F}$. 5

(c) (i) Express $\vec{\nabla}\times\vec{A}$ and $\nabla^2\psi$ in spherical coordinates. 2+3=5

(ii) Find the element of arc length on a sphere of radius a. 5

GROUP—B

(Properties of Matter)

(*Marks* : 25)

5. Answer the following questions : 1×4=4

(a) Write the expression for Young's modulus, when increase in length is not proportional to applied force.

(b) Draw the stress-strain graph for a wire.

(c) What is the cause of surface tension of a liquid?

(d) What will happen to angle of contact of a liquid, when the temperature increases?

6. Answer the following questions : 2×3=6

(a) The volume of a solid does not vary with pressure. Find Poisson's ratio for the solid.

(b) Distinguish between wave and ripple.

(c) How does the viscosity of liquids and gases vary with temperature?

7. Answer any *one* of the following questions : 5

(a) (i) Show that tensile strain in a filament is directly proportional to its distance from the neutral axis. 3

(ii) A steel wire of length 2 m is stretched through 2 mm. The cross-sectional area of the wire is 40 mm^2. Calculate the elastic potential energy stored in the wire in the stretched condition. Young's modulus of steel $= 2 \times 10^{11}$ N/m^2. 2

(b) (i) Write down the limitations of Poiseuille's formula for the rate of flow of liquid through a capillary tube. 3

(ii) In the Poiseuille experiment, the following observations were made :

Volume of water collected in 5 minutes = 40 c.c.

Head of water = 0·4 m

Length of capillary tube = 0·602 m

Radius of capillary tube $= 0 \cdot 52 \times 10^{-3}$ m

Calculate the coefficient of viscosity of water. 2

8. Answer *either (a) or (b)* : 10

(a) (i) Derive an expression for the twisting couple per unit angular twist for a solid cylinder.

Using the above relation, find the twisting couple per unit twist for hollow cylinder. 5+2=7

(ii) Explain with reason, why a hollow cylinder is stronger than a solid cylinder of same length, mass and material. 3

(b) *(i)* Show that the excess pressure acting on a curved surface of a curved membrane is given by

$$P = 2T\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

where r_1 and r_2 are the radii of curvature and T is the surface tension of the membrane.

Using the above relation, calculate the excess pressure for cylindrical film. 5+2=7

(ii) Two soap bubbles of radii a and b coalesce to form a single bubble of radius c. If the external pressure is P, show that the surface tension is given by

$$S = \frac{P(c^3 - a^3 - b^3)}{4(a^2 + b^2 - c^2)}$$ 3

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