2018

PHYSICS

(Major)

Paper : 1.1

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

GROUP—A

(Mathematical Methods)

(*Marks* : 20)

1. *(a)* Find the Cartesian component of a vector $\vec{C}$ which is perpendicular to the vector $\vec{A}$ and vector $\vec{B}$, where

$$\vec{A} = 2\hat{i} - \hat{j} + \hat{k} \quad \text{and} \quad \vec{B} = 3\hat{i} + 4\hat{j} - \hat{k}$$ 1

(b) Define vector field in a region of space. Give an example of vector field. 1

2. *(a)* Give the vector diagram representation of $\vec{A} \times \vec{B} = \vec{C}$ and $\vec{B} \times \vec{A} = \vec{D}$. Name a physical vector quantity which is the product of two vectors. 2

(b) What is the physical significance of divergence of a vector? 2

(c) Find the projection of vector $\vec{A}$ on vector $\vec{B}$, where $\vec{A} = 3\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{B} = \hat{i} - 3\hat{j} + 4\hat{k}$. 2

(d) A particle with position vector $\vec{r} = \cos \omega t \hat{i} + \sin \omega t \hat{j}$ moves with a constant angular velocity ω. The linear velocity $\vec{v}$ of the particle is perpendicular to $\vec{r}$. Show that $\vec{r} \times \vec{v}$ is a constant vector. 2

3. Answer any *two* questions : 5×2=10

(a) If $\overline{V} = \overline{a} \cos \omega t + \overline{b} \sin \omega t$, find that

$$\overline{V} \times \frac{d\overline{V}}{dt} = \omega(\vec{a} \times \vec{b})$$

Here $\vec{a}$ and $\vec{b}$ are two constant non-linear vectors and ω is constant scalar.

(b) If $r = (x^2 + y^2 + z^2)^{1/2}$, show that

$$\nabla^2\left(\frac{1}{r}\right) = 0$$

(c) Show that gradient of any scalar field $\phi(r)$ is irrotational and the curl of any vector field $\vec{V}(r)$ is solenoidal.

GROUP—B

(Mechanics)

(*Marks* : 40)

4. (a) What is fictitious force? Give an example of it. 1

(b) Is the centre of mass frame of reference an inertial frame? Explain. 1

(c) A particle is moving horizontally at the equator. What is the value of Coriolis force acting on it in local coordinate system? 1

(d) What is the difference between laboratory frame of reference and centre of mass frame of reference? 1

(e) When is a force field said to be conservative? Give an example of conservative force. 1

(f) Can we have equipotential surfaces of the gravitational field of a point mass? What is the value of work done if a mass moves on an equipotential surface? 1

5. (a) Two particles of mass 2 kg each are moving with velocity $(2\hat{i}+4\hat{j})$ m/s and $(5\hat{i}+6\hat{j})$ m/s respectively. Find the kinetic energy of the system relative to centre of mass. 2

(b) Show that force field given by $\vec{F} = x^2yz\hat{i} - xyz^2\hat{k}$ is non-conservative. 2

6. Answer any *two* questions : 5×2=10

(a) Show that whenever a body is acted upon by a number of forces such that the resultant is not zero, then the work done by the resultant force is equal to the change in the kinetic energy of the body.

(b) Calculate the moment of inertia of a thin hollow sphere about its diameter.

(c) Find the centre of mass of a uniform solid hemisphere of radius a.

7. Answer any *two* questions : 10×2=20

(a) (i) Distinguish between inertial mass and gravitational mass.

(ii) Obtain an expression for the gravitational potential and field due to a thin uniform spherical shell at an external point.

(iii) The radius of the earth is $6{\cdot}637 \times 10^6$ m and its mean density is $5{\cdot}57 \times 10^3$ kg/m^3. Calculate earth surface potential. Given $G = 6{\cdot}66 \times 10^{11}$ Nm^2 kg^{-2}. 2+5+3=10

(b) (i) What is the effect of Coriolis force on a particle falling freely under the action of gravity?

(ii) Show that the angular accelerations of a particle in a fixed system and a rotating system are same. 5+5=10

(c) (i) Give a schematic diagram of elastic collision of two particles in centre of mass frame and laboratory frame.

(ii) Obtain a relation of scattering angles in these two frames of reference. 2+8=10

(d) (i) Prove that a conservative force can be expressed as negative gradient of potential.

(ii) Two particles of masses m_1 and m_2 separated by infinite distance apart, attract each other according to the law of gravitation. Considering the particles to be initially at rest, show that their velocity of approach

$$v = \sqrt{\frac{2G(m_1 + m_2)}{a}}$$

where a is final separation of the two masses.

(iii) Find the force field associated with the potential energy $V = Ae^{\alpha(x+y+z)}$, where A and α are constants. 4+4+2=10

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