2018

MATHEMATICS

(Major)

Paper : 2.2

(Differential Equation)

Full Marks : 80

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following as directed : 1×10=10

(a) Write down the degree of the differential equation

$$y = \sqrt{x}\frac{dy}{dx} + \frac{k}{\frac{dy}{dx}}$$

(b) Is the differential equation

$$\frac{dy}{dx} = \frac{y}{x} + \sin\frac{y}{x}$$

homogeneous?

(c) What is the integrating factor of the differential equation $x\,dy - y\,dx = 0$?

(d) Write the particular integral of the differential equation

$$(D^2 - 3D + 2)y = e^{5x}$$

(e) What do you mean by trajectory of a given family of curves?

(f) Write down the general solution of the differential equation

$$y = px + e^p ; \quad p = \frac{dy}{dx}$$

(g) Write the conditions for exactness of a total differential equation

$$P\,dx + Q\,dy + R\,dz = 0$$

(h) The partial differential equations can be formed by the elimination of

(i) arbitrary constants only

(ii) arbitrary functions only

(iii) arbitrary functions and arbitrary constants

(iv) None of the above

(Choose the correct answer)

(i) Write the standard form of the linear partial differential equation of order one.

(j) Write down Lagrange's auxiliary equations of the linear partial differential equation

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = xyz$$

2. Answer the following questions : 2×5=10

(a) Find the differential equation of all straight lines passing through the origin.

(b) Solve :

$$\cos(x+y)\,dy = dx$$

(c) Solve :

$$\frac{d^3y}{dx^3} - 2\frac{d^2y}{dx^2} + 4\frac{dy}{dx} - 8y = 0$$

(d) Solve :

$$yz\,dx + 2zx\,dy - 3xy\,dz = 0$$

(e) Construct the partial differential equation by eliminating a and b from

$$z = ax + (1-a)y + b$$

3. Answer any *four* parts : 5×4=20

(a) Solve :

$$x^2\left(\frac{dy}{dx}\right)^2 + xy\frac{dy}{dx} - 6y^2 = 0$$

(b) Show that the system of confocal and coaxial parabolas $y^2 = 4a(x+a)$ is self-orthogonal, a being parameter.

(c) Solve :

$$(D^2 - 2D + 4)y = e^x \cos x$$

(d) Solve :

$$x^3 \frac{d^3y}{dx^3} + 2x^2 \frac{d^2y}{dx^2} + 2y = 10\left(x + \frac{1}{x}\right)$$

(e) Apply variation of parameters to solve the differential equation

$$\frac{d^2y}{dx^2} + 4y = 4 \tan 2x$$

(f) Solve :

$$\frac{dx}{dt} - 7x + y = 0; \quad \frac{dy}{dt} - 2x - 5y = 0$$

4. Answer *either (a)* and *(b) or (c)* and *(d)* : 5+5=10

(a) Show that the differential equation

$$\frac{dy}{dx} + \frac{ax + hy + g}{hx + by + f} = 0$$

is exact and hence solve it.

(b) Write down Bernoulli's differential equation. Solve the following differential equation by reducing it in linear form :

$$\frac{dy}{dx} + xy = xy^2$$

(c) Reduce the differential equation

$$(y + xp)^2 = x^2 p$$

where $p = \frac{dy}{dx}$ to Clairaut's form by substituting $xy = v$ and hence solve the equation.

(d) Solve

$$\frac{d^2y}{dx^2} + y = 0$$

given $y = 2$ for $x = 0$; $y = -2$ for $x = \frac{\pi}{2}$.

5. Answer *either (a)* and *(b) or (c)* and *(d)* : 5+5=10

(a) Solve

$$\sin^2 x\left(\frac{d^2y}{dx^2}\right) = 2y$$

given $y = \cot x$ is a solution.

(b) Find $f(z)$ such that the equation

$$\left(\frac{y^2+z^2-x^2}{2x}\right)dx - y\,dy + f(z)\,dz = 0$$

is integrable. Hence solve it.

(c) Solve :

$$x\frac{d^2y}{dx^2} - (2x-1)\frac{dy}{dx} + (x-1)y = 0$$

(d) Solve :

$$\frac{dx}{x(y^2-z^2)} = \frac{dy}{y(z^2-x^2)} = \frac{dz}{z(x^2-y^2)}$$

6. Answer *either* (a) and (b) *or* (c) and (d) : 5+5=10

(a) Solve :

$$xz^3\,dx - z\,dy + 2y\,dz = 0$$

(b) Solve

$$x^4\frac{d^2y}{dx^2} + 2x^3\frac{dy}{dx} + x^2y = 0$$

by changing the independent variable x to z.

(c) Reduce the differential equation

$$\frac{d^2y}{dx^2} - \frac{2}{x}\frac{dy}{dx} + \left(1+\frac{2}{x^2}\right)y = xe^x$$

to its normal form and hence solve it.

(d) Derive the partial differential equation by elimination of arbitrary function ϕ from the equation $\phi(u, v) = 0$, where u and v are functions of x, y and z.

7. Answer *either (a)* and *(b)* or *(c)* and *(d)* : 5+5=10

(a) Solve by Lagrange method :

$$z(xp - yq) = y^2 - x^2$$

(b) Find the integral surface of the partial differential equation

$$(x - y)p + (y - x - z)q = z$$

through the circle $z = 1,\ x^2 + y^2 = 1$.

(c) Solve by Charpit's method :

$$(p^2 + q^2)y = qz$$

(d) Find the complete integral of

$$q^2 = z^2 p^2 (1 - p^2)$$

Find also the singular integral, if it exists.

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