Electric field due to a dipole at any given point

$P\cos\theta \rightarrow$ component of dipole moment $\vec{P}$ along OP

$P\sin\theta \rightarrow$ component of dipole moment $\vec{P}$ in $\perp^r$ direction of OP

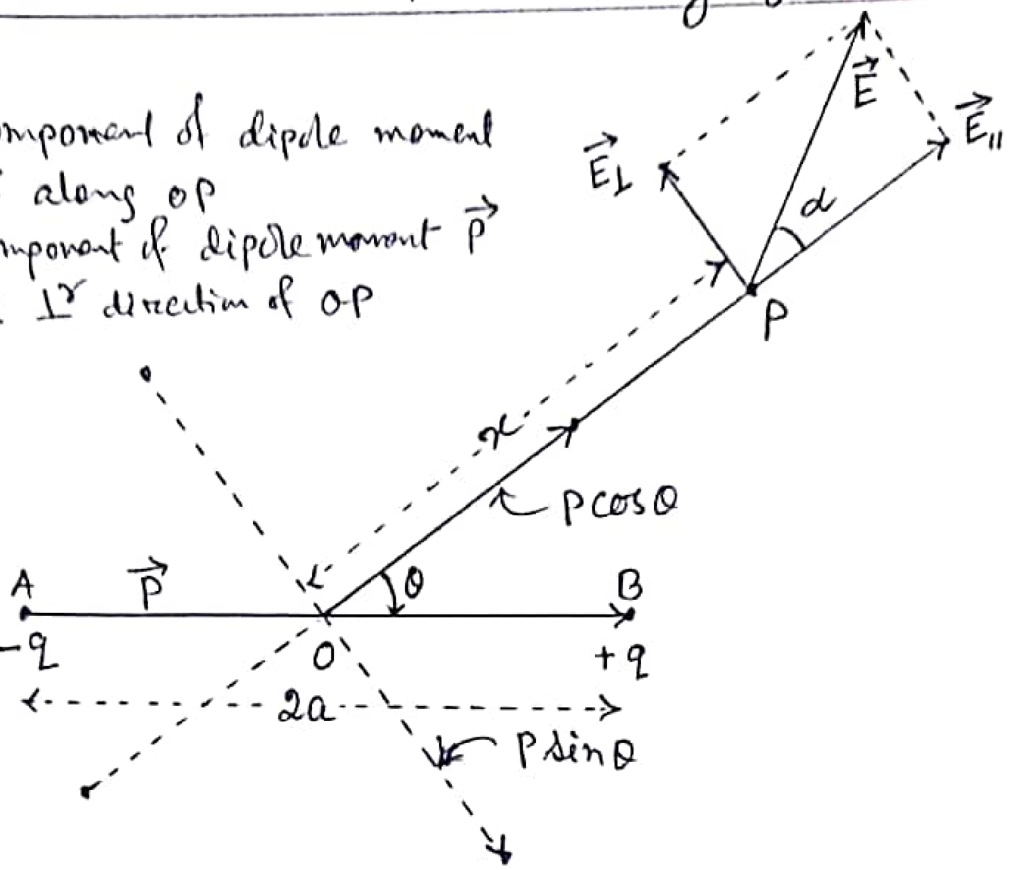

Fig. 1: Electric field at pt. P

Electric field at P due to $P\cos\theta$ is

$$E_{\parallel} = \frac{1}{4\pi\epsilon_0} \frac{2P\cos\theta}{x^3}, \text{ along OP}$$

and due to $P\sin\theta$ is

$$E_{\perp} = \frac{1}{4\pi\epsilon_0} \frac{P\sin\theta}{x^3}, \text{ along } PE_{\perp}$$

$\therefore$ Net field at P is

$$\vec{E} = \vec{E}_{\parallel} + \vec{E}_{\perp}$$

$$\Rightarrow E = \sqrt{E_{\parallel}^2 + E_{\perp}^2 + 2E_{\parallel}E_{\perp}\cos 90^\circ} = \sqrt{E_{\parallel}^2 + E_{\perp}^2}$$

$$= \sqrt{\left(\frac{1}{4\pi\epsilon_0}\frac{P}{x^3}\right)^2 4\cos^2\theta + \left(\frac{1}{4\pi\epsilon_0}\frac{P}{x^3}\right)^2 \sin^2\theta}$$

$$= \frac{1}{4\pi\epsilon_0}\frac{P}{x^3}\sqrt{4\cos^2\theta + \sin^2\theta}$$

$$= \frac{1}{4\pi\epsilon_0} \frac{P}{x^3} \sqrt{3\cos^2\theta + 1} \quad (\because \sin^2\theta = 1 - \cos^2\theta)$$

$$\Rightarrow \boxed{E = \frac{1}{4\pi\epsilon_0} \frac{P\sqrt{3\cos^2\theta + 1}}{x^3}} \rightarrow \text{magnitude}$$

Again, $\tan\alpha = \dfrac{E_\perp}{E_\parallel} = \dfrac{\frac{1}{4\pi\epsilon_0} \frac{P\sin\theta}{x^3}}{\frac{1}{4\pi\epsilon_0} \frac{2P\cos\theta}{x^3}}$

$$\Rightarrow \tan\alpha = \frac{1}{2}\tan\theta$$

$$\therefore \boxed{\alpha = \tan^{-1}\left(\frac{1}{2}\tan\theta\right)} \rightarrow \text{direction}$$

———— X ————